

# GSI Oscillations as Interference of Neutrino Flavour Mass–Eigenstates and Measuring Process

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This paper is addressed to the proof of the important role of measuring apparatus, i.e. the measuring process, in the formation of *necessary* and *sufficient* conditions for the explanation of a time modulation of K–shell electron capture (EC) decay rates of hydrogen–like (H–like) heavy ions (or the GSI oscillations) as the interference of neutrino mass–eigenstates of the electron neutrino constituents. For our analysis we use a toy–model, which has been recently proposed by Peshkin arXiv: 1403.4292 [nucl-th] for a verification of the mechanism of the GSI oscillations as the interference of neutrino mass–eigenstates by Ivanov and Kienle Phys. Rev. Lett. **103**, 062502 (2009).

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The GSI oscillations that is an unexpected time modulation of the K–shell electron capture (EC) decay rates of the hydrogen–like (H–like) ions have been observed at GSI Darmstadt in [1]–[4] and confirmed recently in [5]. In the GSI experiments on the EC decays  $p \rightarrow d + \nu_e$ , where  $p$  and  $d$  are the parent H–like and daughter ions in their ground states and  $\nu_e$  is the electron neutrino, the rates of the number of daughter ions  $N_d(t)$  and the number of parent ions  $N_p(t)$  at time  $t$  after the injection into the experimental storage ring (ESR) are related by the equation

$$\frac{dN_d(t)}{dt} = \lambda_{\text{EC}} (1 + a \cos(\omega t + \phi)) N_p(t), \quad (1)$$

where  $\lambda_{\text{EC}}$  is the EC decay constant. The term  $a \cos(\omega t + \phi)$  defines an unexpected time modulation or the GSI oscillations with an amplitude  $a$ , a period  $T = 2\pi/\omega$  and a phase–shift  $\phi$ . In turn, as has been reported in [3, 4] and confirmed in [5] the rates of the number of daughter ions  $d'$  of the  $\beta^+$ –decays  $p \rightarrow d' + e^+ + \nu_e$  of the H–like heavy ions  $p$ , measured at the same conditions as the EC decays  $p \rightarrow d + \nu_e$ , do not show a time modulation.

As has been proposed in [6]–[11], the time modulation of the EC decay rates and its unobservability in the  $\beta^+$ –decay rates of the H–like heavy ions can be explained by the interference of neutrino mass–eigenstates, which are constituents of the electron neutrino defining the wave function of the electron neutrino in the form of the superposition  $|\nu_e\rangle = \sum_{j=1}^{N_\nu} U_{ej}^* |\nu_j\rangle$ , where  $U_{ej}^*$  are the matrix elements of the  $N_\nu \times N_\nu$  mixing matrix and  $N_\nu \geq 3$  is the number of neutrino mass–eigenstates  $|\nu_j\rangle$  with masses  $m_j$  [12]. In such an approach the EC decays  $p \rightarrow d + \nu_e$  are defined by the decay channels  $p \rightarrow d + \nu_j$  ( $j = 1, 2, \dots, N_\nu$ ), which may interfere at certain conditions, caused by interactions of ions with measuring apparatus in the ESR, and show the time modulation with frequencies proportional to  $(\Delta m_{ij}^2)_{\text{GSI}}/2M_p$ , where  $(\Delta m_{ij}^2)_{\text{GSI}} = \tilde{m}_i^2 - \tilde{m}_j^2$  are the differences of squared dynamical masses  $\tilde{m}_i$  and  $\tilde{m}_j$  of neutrino mass–eigenstates  $|\nu_i\rangle$  and  $|\nu_j\rangle$ , respectively, [6, 9] and  $M_p$  is the parent ion mass. As has been shown in [9], neutrino mass–eigenstates in the weak decays of highly charged heavy ions can acquire mass–corrections, caused by polarisation of virtual  $\nu_j \rightarrow \sum \ell^- W^+ \rightarrow \nu_j$  pairs in the strong Coulomb fields of daughter ions, where  $\ell^-$  and  $W^+$  are leptons and the  $W^+$ –boson of electroweak interactions, respectively. These mass corrections increase the differences  $(\Delta m_{ij}^2)_{\text{GSI}}$ , extracted from periods of the GSI oscillations, in comparison to the differences  $\Delta m_{ij}^2 = m_i^2 - m_j^2$  of squared *bare* masses  $m_j$  of neutrino mass–eigenstates [12]. In contrast to the EC decay rates the interference of the decay channels  $p \rightarrow d' + e^+ + \nu_i$  and  $p \rightarrow d' + e^+ + \nu_j$  ( $i \neq j \in 1, 2, \dots, N_\nu$ ) of the  $\beta^+$ –decays  $p \rightarrow d' + e^+ + \nu_e$  occurs with much higher frequencies  $(\Delta m_{ij}^2)_{\text{GSI}}/2Q_{\beta^+}$  [10], where  $Q_{\beta^+}$  are the Q–values of the  $\beta^+$  decays, which are of order of a few MeV. As a result such a time modulation is not observable in the GSI experiments [3–5].

Following [6]–[11] one may state that i) the *necessary* and *sufficient* condition for the interference of neutrino mass–eigenstates is violation of energy and 3–momentum in the EC decays, caused by interactions of ions in the ESR with measuring apparatus or by the measuring process [13], and ii) the orthogonality  $\langle \nu_i | \nu_j \rangle \sim \delta_{ij}$  of the final states of

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the decay channels  $p \rightarrow d + \nu_i$  and  $p \rightarrow d + \nu_j$  ( $i \neq j \in 1, 2, \dots, N_\nu$ ) of the EC decays  $p \rightarrow d + \nu_e$  does not prevent from the interference of neutrino mass-eigenstates.

Recently a toy-model for a verification of our explanation of the GSI oscillations by means of the interference of neutrino mass-eigenstates has been proposed by Murray Peshkin [14]. Within such a toy-model Peshkin confirms our assertion that the orthogonality of the final states of the decay channels  $p \rightarrow d + \nu_i$  and  $p \rightarrow d + \nu_j$  with  $i \neq j \in 1, 2, \dots, N_\nu$  of the EC decay  $p \rightarrow d + \nu_e$  makes no influence on the suppression of the interference between neutrino mass-eigenstates. However, his analysis of the suppression of the interference of neutrino mass-eigenstates due to the absence of off-diagonal matrix elements in the definition of the survival probability of the parent ions suffers from the problem of the lack of measuring apparatus and interactions of parent and daughter ions with measuring apparatus, i.e. the measuring process [13].

Let us repeat shortly Peshkin's analysis of our approach to the GSI oscillations. According to Peshkin [14], the Hilbert space of the vector states of the decaying system consists of the parts in which the parent  $p$  ion and decay products  $d\nu_1$  and  $d\nu_2$  are present, where  $d$  is a daughter ion and  $|\nu_j\rangle$  are neutrino mass-eigenstates with masses  $m_j$  for  $j = 1, 2$ . Then, let  $|\psi(t)\rangle$  be the vector state of the entire system and let  $P_p$  be the projection operator on the parent ion state  $|P_p\psi(t)\rangle$ . The translational invariant Hamilton operator of the system is given by  $H = H_0 + V$ , where  $H_0$  is the Hamilton operator of the free objects  $p$ ,  $d$  and  $\nu_j$  for  $j = 1, 2$ , whereas  $V$  describes the interaction between them. The survival probability  $S(t)$  of the parent ion at time  $t$  after the injection into the ESR is defined by [14]

$$S(t) = \langle P_p\psi(t) | P_p\psi(t) \rangle = \langle \psi(0) | e^{+iHt} P_p e^{-iHt} | \psi(0) \rangle. \quad (2)$$

The rate  $R(t)$  of the survival probability is given by

$$R(t) = -\frac{dS(t)}{dt} \propto 1 + a \cos(\omega t + \phi), \quad (3)$$

where the time modulated term  $a \cos(\omega t + \phi)$  can appear only if the interference of the decay channels  $p \rightarrow d + \nu_1$  and  $p \rightarrow d + \nu_2$  or neutrino mass-eigenstates  $|\nu_1\rangle$  and  $|\nu_2\rangle$  is allowed.

For the analysis of the conditions, at which the interference between the decay channels  $p \rightarrow d + \nu_1$  and  $p \rightarrow d + \nu_2$  can occur, Peshkin transcribes the survival probability into the form

$$S(t) = \langle \psi(0) | e^{+iHt} P_p e^{-iHt} | \psi(0) \rangle = \int d^3K' d^3K \langle \psi(0) | \vec{K} \rangle \langle \vec{K} | e^{+iHt} P_p e^{-iHt} | \vec{K}' \rangle \langle \vec{K}' | \psi(0) \rangle, \quad (4)$$

where  $\vec{K}$  and  $\vec{K}'$  are total 3-momenta and other dynamical variables accompanying  $\vec{K}$  and  $\vec{K}'$  are assumed [14]. The interference of the decay channels  $p \rightarrow d + \nu_1$  and  $p \rightarrow d + \nu_2$  or the time modulation can appear in the survival probability  $S(t)$  only due to off-diagonal matrix elements  $\langle \vec{K} | e^{+iHt} P_p e^{-iHt} | \vec{K}' \rangle \neq 0$  [14]. However, as has been pointed out by Peshkin, the matrix elements  $\langle \vec{K} | e^{+iHt} P_p e^{-iHt} | \vec{K}' \rangle$  should be diagonal

$$\langle \vec{K} | e^{+iHt} P_p e^{-iHt} | \vec{K}' \rangle \sim \delta^{(3)}(\vec{K} - \vec{K}') \langle \vec{K} | e^{+iHt} P_p e^{-iHt} | \vec{K} \rangle, \quad (5)$$

since the Hamilton operator  $H$  is translational invariant. Peshkin also assumes that off-diagonal matrix elements can appear due to external fields such as magnetic fields, stabilising a motion of ions in the ESR, and electron cooling in the ESR. However, such a possibility has been rejected by Peshkin, since in this case parameters of a time modulation should depend on parameters, characterising these fields, and a frequency of a time modulation may or may not depend on  $\Delta m_{21}^2 = m_2^2 - m_1^2$ , where  $m_2$  and  $m_1$  are the masses of neutrino mass-eigenstates  $|\nu_2\rangle$  and  $|\nu_1\rangle$ , respectively.

Below we show that Peshkin's analysis of off-diagonal matrix elements of the survival probability  $S(t)$ , based on the use of the translational invariant Hamilton operator  $H$  for the description of evolution of the decaying system and its decay products in the ESR, is not complete and interactions of ions with measuring apparatus, i.e. the measuring process, should be taken into account [13]. Indeed, apart from the magnetic fields, stabilising a motion of ions in the ESR after the injection of parent ions into the ESR and cooling, that reduces a relative transverse velocity spread to  $\Delta v/v \approx 5 \times 10^{-7}$  [5], ions are being monitored by both the resonant pickup (the 245 MHz resonator) and the capacitive pickup (the Schottky noise detector) during an optimised measuring period 54 s with sampling times  $\delta t = 64$  ms and  $\delta t = 32$  ms, respectively [5]. This implies that evolution of ions in the ESR cannot be analysed correctly if interactions of ions with measuring apparatus is switched off, i.e. using only the Hamilton operator  $H$ .

The account for interactions between measuring apparatus and a quantum system leads to a change of its wave function. According to Wigner [13], von Neumann [15] and London and Bauer [16] (see also [17] and [18–20]), wave functions of quantum systems, coupled to measuring apparatus, evolve in time with i) unitary Schrödinger evolution between measurements and ii) sudden and non-unitary evolution at time of each measurement. Moreover for sequence of measurements, distributed in time, measurements can overlap and interleave in a complicated way [19]. As a result,

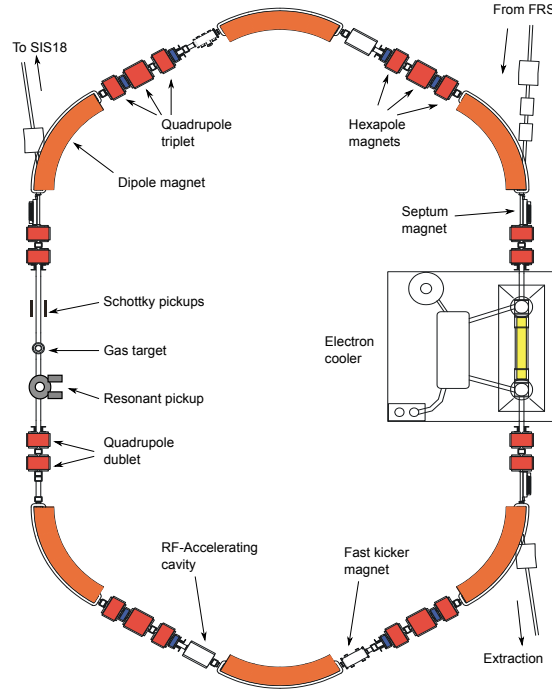


FIG. 1: The ESR at GSI, equipped by the resonant and capacitive pickups detecting parent and daughter ions [22].

there can be in general no time, when quantum systems are undisturbed by measurements, and, correspondingly, no time intervals of unitary evolution [19].

How such a dynamics of evolution of wave functions of highly charged heavy ions, coupled to measuring apparatus, is realised in GSI experiments? In the ESR ions, moving after cooling with a velocity  $v = 0.71$ , make a total revolution over the circumference of the ESR with the length  $\Pi = 108.36$  m for  $T_{\text{rev}} = 510$  ns. During the measurement time  $t \leq 54$  s ions make enormous number of total revolutions, going during every revolution through the resonant and capacitive pickups, located at the ESR as it is shown in Fig. 1.

Passing through the resonant and capacitive pickups ions lose and gain energy [22]. Such a process has a random and cumulative character, which leads to energy violation and to disturbance of wave functions of ions. For example, in the resonant pickup the interaction of ions with only the fundamental mode leads to an energy loss of order  $\Delta\mathcal{E} \sim 15 \mu\text{eV}$  [22]. Such an energy loss corresponds to an observation time  $\Delta t \sim 2\pi/\Delta\mathcal{E} \sim 0.28$  ns [15, 23], during which ions are observed continuously [22]. Thus, during the observation time  $\Delta t \sim 0.27$  ns, caused by the interaction with only the fundamental mode of the resonant pickup, wave functions of ions evolve according to non-unitary evolution. In the capacitive pickup ions pass through a pair of parallel metal plates and induce images of charges on the surface of each plate [24]. The signals coming from the two plates are amplified and used in dependence on specific purposes [22]. Interactions of ions with their images and plates lead to energy fluctuations and to disturbance of wave functions ions and, correspondingly, to non-unitary evolution. It is obvious that wave functions of ions cannot be restored to their undisturbed form immediately after the exit from the region of the resonant and capacitive pickups and there is a certain retardation of such a restoration, which, of course, becomes slower and slower with the number of revolutions of ions over the circumference of the ESR. As a result, the resonant and capacitive pickups divide the ESR into the regions of non-unitary and unitary evolution of wave functions of ions. Such a decomposition of the ESR is not stationary. Time intervals of non-unitary and unitary evolution increase and decrease, respectively. Of course, a detection of ions by both the resonant and capacitive pickups does not destroy the quantum states of H-like heavy ions, since energy fluctuations are much smaller then binding energies, but causes a smearing of energies and momenta of daughter ions  $d$  leading to indistinguishability of the decay channels  $p \rightarrow d + \nu_1$  and  $p \rightarrow d + \nu_2$  and, correspondingly, indistinguishability of the decay channels  $p \rightarrow d + \nu_1$  and  $p \rightarrow d + \nu_2$  of the EC decay  $p \rightarrow d + \nu_e$  leading to their interference [6]–[11]. As has been shown in [6]–[11], indistinguishability of daughter ions and the decay channels  $p \rightarrow d + \nu_1$  and  $p \rightarrow d + \nu_2$  can be qualitatively described in terms of Heisenberg's uncertainty relations.

Hence, for the correct analysis of the origin of the interference of neutrino mass-eigenstates the Hamilton operator of Peshkin's toy-model should be taken in the form  $H_{\text{tot}} = H + U(t)$  [18–20, 23], where  $U(t)$  is a time-dependent and translational non-invariant potential of ions coupled to measuring apparatus, i.e. the resonant and capacitive pickups.

The potential  $U(t)$  does not vanish only during time intervals of observation of parent and daughter ions, coupled to the resonant and capacitive pickups [23]. A time evolution of the Hamilton operator  $H$  and the 3-momentum operator  $\vec{K}$  of parent ions and decay products is defined by the Heisenberg equations of motion [20]

$$\frac{dH}{dt} = i[U(t), H] \quad , \quad \frac{d\vec{K}}{dt} = i[U(t), \vec{K}]. \quad (6)$$

Since  $[U(t), H] \neq 0$  and  $[U(t), \vec{K}] \neq 0$ , the Heisenberg equations of motion testify that energy and momentum in the EC decays are violated [25]. This results in non-vanishing off-diagonal matrix elements in Eq.(4) and provides the basis for the time modulation of the survival probability  $S(t)$  and its rate  $R(t)$ . In other words violation of energy and momentum in the EC decays, caused by interactions of parent and daughter ions with measuring apparatus, defines *necessary* and *sufficient* conditions for the appearance of the time modulation of the EC decay rate  $p \rightarrow d + \nu_e$  as the interference of the decay channels  $p \rightarrow d + \nu_1$  and  $p \rightarrow d + \nu_2$  or neutrino mass-eigenstates  $|\nu_1\rangle$  and  $|\nu_2\rangle$  [11]. A proportionality of the modulation frequency to  $\Delta m_{21}^2/2M_p = (m_2^2 - m_1^2)/2M_p$  is justified by non-conservation and smearing of 3-momenta  $\vec{q}_1$  and  $\vec{q}_2$  of the daughter ions in the decay channels  $p \rightarrow d + \nu_1$  and  $p \rightarrow d + \nu_2$  around a 3-momentum  $\vec{q} \pm \delta\vec{q}$ , where  $\delta\vec{q}$  is uncertainty of 3-momentum of daughter ions caused by the measuring process [6–8, 11].

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